Infrared Modification of Gravity[*]

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Abstract

In this lecture I address the issue of possible large distance modification of gravity and its observational consequences. Although, for the illustrative purposes we focus on a particular simple generally-covariant example, our conclusions are rather general and apply to large class of theories in which, already at the Newtonian level, gravity changes the regime at a certain very large crossover distance r_c. In such theories the cosmological evolution gets dramatically modified at the crossover scale, usually exhibiting a "self-accelerated" expansion, which can be differentiated from more conventional "dark energy" scenarios by precision cosmology. However, unlike the latter scenarios, theories of modified-gravity are extremely constrained (and potentially testable) by the precision gravitational measurements at much shorter scales. The reason is that modification implies the new propagating light degrees of freedom (additional polarizations of graviton) which penetrate at short distances in a rather profound way, and lead to the deviations from Einstein's gravity at the source-dependent scales. Despite the presence of extra polarizations of graviton, the theory is compatible with observations, since the naive perturbative expansion in Newton's constant breaks down at a certain intermediate scale. This happens because the extra polarizations have couplings singular in $1/r_c$. However, the correctly resummed non-linear solutions are regular and exhibit continuous Einsteinian limit. Contrary to the naive expectation, explicit examples indicate that the resummed solutions remain valid after the ultraviolet completion of the theory, with the loop corrections taken into account.

1   Introduction and Framework.

The question that I will try to address in this talk is whether it is possible to self-consistently modify General Relativity (GR) in far infrared (IR), and whether such a modification could come from some underlying fundamental theory. Under the self-consistent modification I mean modification that is generally-covariant, and is free of unphysical negative or complex norm states (i.e., is ghost-free). I shall only be interested in theories that are not reducible to a simple addition (to the ordinary gravity) of new light scalar or vector degrees of freedom.

Such theories are motivated by the "Dark Energy" and the "Cosmological Constant" problems. However, even if one still hopes to find a less radical solution to these problems, the question of self-consistent IR-modification of gravity is a valid fundamental question.

As we shall see, the answer to this question is probably "yes", as there is at least one existence proof for such modifications (and probably there are many others). However, the theories in question are extremely constrained, because any modification of gravity (respecting locality and causality) in arbitrary far IR implies existence of new gravitational degrees of freedom (new polarizations of graviton) that inevitably penetrate at short distances and lead to the potentially-observable effects.

If there is some new gravitational dynamics in far IR, in an effective short-distance action it would manifest itself in form of some effectively-non-local operators (possibly the infinite series) that should dominate over the standard Einstein-Hilbert action at large distances, or equivalently at small curvatures. Such operators should be suppressed by a crossover scale (r_c) beyond which the new dynamics takes over and gravity changes the regime.

$$\int d^4x \sqrt{-g}\, M_P^2\, R\, +\, [\text{something dominating at } r > r_c]. \tag{1}$$

One could naively think that at short distances $r \ll r_c$, the effect of the new terms can be made arbitrarily small. However, this is not the case. General Relativity is an extremely constrained theory. Requirement of the massless graviton plus general-covariance uniquely fixes the low energy action to be the Einstein-Hilbert action. Hence, any deviation from it implies that gravity can no longer be mediated by a massless spin-2 particle, but should inevitably include extra states.

I shall discuss the above features on an example of a simple generally-covariant theory that modifies gravity at large distances, however, our qualitative conclusions will be very general and apply to large class of IR-modified theories (at least the ones that at the linearized level admit a sensible spectral representation). The two emerging features of such IR-modified theories are the (expected) strong modification of the cosmological evolution at the scale r_c, and also a strong modification of laws of gravity at a *source-dependent* scale r_*. [1]   Depending on a particular

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[1]In a sense this is one and the same feature, since for the Universe $r_* = r_c$ (see below).

gravitating source, the scale r_* can be small enough so that the deviations can be potentially detectable by precision gravitational experiments at much shorter (e. g., solar system) distances.

The model in question, is a brane-world model of[1], which consist of a brane embedded in five-dimensional Minkowski space. I shall use the notion of a brane in a very general sense of a 4D sub-space on which the standard model particles live. The action of the theory is

$$S = \frac{M_P^2}{4r_c} \int d^4x \, dy \, \sqrt{|g^{(5)}|} \, R_5 + \frac{M_P^2}{2} \int d^4x \, \sqrt{|g|} \, R \, . \tag{2}$$

The first term is just an usual 5D Einstein action, with the five dimensional Plank mass given by $M_*^3 = \frac{M_P^2}{4r_c}$. $g_{\mu\nu}(x)$ is the induced metric on the brane, which in the approximation of the "rigid" brane is simply given by the value of the five-dimensional metric at the position of the brane. For instance, if we locate the brane at the origin of the fifth coordinate y, the induced metric takes the form $g_{\mu\nu}(x) = g_{\mu\nu}^{(5)}(x, y = 0)$. The 4D Einstein term on the brane (R) plays the crucial role in generating 4D gravity on the brane at intermediate distances, despite the fact that the space is actually five-dimensional. Here is how this effect comes about. Consider a gravitating source localized on the brane, and let us ask what kind of gravitational field will it create on the brane. That is, what type of a Newtonian attractive force will it exert on the test bodies that are also localized on the brane? The Newtonian force is mediated by the virtual graviton exchange, which in our case (after some gauge fixing) satisfy the following linearized equation

$$\left(\frac{1}{r_c}(\nabla^2 - \partial_y^2) + \delta(y)\nabla^2 \right) h_{\mu\nu} = - \frac{1}{M_P^2} \left\{ T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}T_\alpha^\alpha \right\} \delta(y) + \delta(y)\, \partial_\mu \partial_\nu \, h_\alpha^\alpha \, , \tag{3}$$

where $T_{\mu\nu}$ is the brane-localized energy-momentum source. The unusual thing about this graviton is that its propagation is governed by two kinetic terms. The 4D kinetic term localized on the brane forces the graviton to propagate according to four-dimensional laws, which would result into the 4D $1/r^2$-Newton's force. The 5D kinetic term, however, allows graviton to also propagate off the brane, and this term alone would of course result into a 5D Newton's force, which scales as $1/r^3$. In the presence of both kinetic terms graviton "compromises", and the force law exhibits the crossover behavior. For $r \ll r_c$ the potential is $1/r$, and for $r \gg r_c$ it turns into $1/r^2$.

The effect of generation of (almost) four-dimensional gravitational interaction at intermediate distances, due to *weakening* of five-dimensional gravity on the brane, we shall call *shielding*[2]. The essence of this effect is that the large kinetic term on the brane makes it hard for a graviton, with the extra momentum exceeding the critical value $1/r_c$, to penetrate the brane. Hence, the number of gravitons that the brane-localized sources are exposed to is much lower then the analogous number for the bulk sources. This creates an effect of a weak 4D-gravity as opposed to strong

5D one. In the Newtonian approximation the effect can be best understood in terms of the four-dimensional mode expansion. From this perspective a high-dimensional graviton represents a continuum of four-dimensional Kaluza-Klein (KK) states and can be expanded in these states. This KK decomposition can schematically be written as follows:

$$h_{\mu\nu}(x,y) \,=\, \int_0^\infty dm\, h_{\mu\nu}^{(m)}(x)\, \sigma_m(y)\,, \tag{4}$$

where $h_{\mu\nu}^{(m)}(x)$ are the four-dimensional spin-2 fields of mass m and $\sigma_m(y)$ are their wave-function profiles in extra dimension. The strength of the coupling of an individual mode of mass m to a brane observer is given by the value of the wave-function at the position of the brane, that is $\sigma_m(0)$. It is a very special m-dependence of this function that makes our model different from simple 5D gravity.

Gravitational force on the brane is mediated by exchange of all the above modes. An each KK-mode gives rise to an Yukawa-type gravitational potential of range m^{-1}, and the net result is

$$V(r) \,\propto\, \frac{1}{M_*^3} \int_0^\infty dm\, |\sigma_m(0)|^2\, \frac{e^{-rm}}{r}\,. \tag{5}$$

So far, this is a generic expression for any 5D gravity theory. At the level of the Newtonian potential, all the difference between our theory and the ordinary 5D gravity is encoded in function $\sigma_m(0)$. In an ordinary five-dimensional gravity we would have $|\sigma_m(0)| = 1$, meaning that all the KK-modes couple to brane-sources with an equal strength. As a result the integral (5) would sum up into the usual 5D potential $\sim 1/M_*^3 r^2$. In our case, because of shielding, the wave-functions of KK gravitons on the brane have the following form [3]

$$|\sigma_m(0)|^2 \,=\, \frac{4}{4 + m^2 r_c^2}\,, \tag{6}$$

which diminishes for $m \gg r_c$, according to the shielding effect. Given this form, it is a trivial exercise to see that the integral (5) exhibits an interpolating behavior between the four and five dimensional regimes at the scale r_c. An equivalent way to interpret the above force is to think of it a being mediated by a four-dimensional resonance of the width $1/r_c$.

2 Implications for the Dark Energy.

Before going to the discussion of more fundamental issues, let me briefly discuss the cosmological implications of IR-modification. Suppose we have an observer, that lives on the brane and knows nothing about the existence of extra dimensions. Based on cosmological observations this observer derives an effective cosmological equation for the four-dimensional scale factor $a(t)$, of the effective 4D Friedmann-Robertson-Walker (FRW) metric. The question is what would be the analog of the

4D Friedmann equation derived in such a way? The answer turns out to be the following (see [4] for details)

$$H^2 \pm H/r_c = \frac{8\pi G_N}{3}\rho \qquad (7)$$

where $H = \dot{a}/a$ is the Hubble parameter, and G_N is the four-dimensional New-ton's constant. The $1/r_c$ correction comes from IR-modification of gravity, due to fifth dimension, and is negligible at early times. So for $H \gg 1/r_c$ the standard FRW cosmology is reproduced. However, for late times modification is dramatic. In particular, the cosmological expansion admits a "self-accelerating" branch with constant $H = 1/r_c$, without need of any matter source[5, 4]. Hence, the large-distance modification of gravity may be a possible explanation for the late time acceleration of the Universe, that is suggested by the current observations[6]. Because of the specific nature of the transition, the upcoming precision cosmological studies can potentially differentiate between the modified gravity and more conventional dark energy scenarios[4, 7].

It is expected that such a late time deSitter phase is a generic property of IR-modified gravity theories, since such modifications should result in non-linearities in H in the modified Friedmann equation, and thus, in general could admit new solutions with constant H, even for $\rho = 0$.

3   Is it Alive?

Let us come back to the gravitational potential and ask the question whether such modification is ruled out. Consider a one-graviton exchange amplitude between the two brane-localized sources $T_{\mu\nu}$ and $T'_{\alpha\beta}$ (the sign tilde denotes the quantities which are Fourier transformed to momentum space)

$$\mathcal{A} = -\frac{8\pi\,G_N}{q^2 + q/r_c}\left(\tilde{T}_{\mu\nu} - \frac{1}{3}\,\eta_{\mu\nu}\,\tilde{T}^\beta_\beta\right)\tilde{T}'^{\mu\nu}. \qquad (8)$$

where $q = \sqrt{q_\mu q^\mu}$ is a four momentum. The analogous amplitude in GR would have the form

$$\mathcal{A}_0 = -\frac{8\pi\,G_N}{q^2}\left(\tilde{T}_{\mu\nu} - \frac{1}{2}\,\eta_{\mu\nu}\,\tilde{T}^\beta_\beta\right)\tilde{T}'^{\mu\nu}. \qquad (9)$$

The two differences are immediately apparent. First, in our IR-modified theory, there is a correction to the denominator that goes as q/r_c. This correction however is unimportant at momenta $q \gg 1/r_c$ (distances $r \ll r_c$). Another difference is the difference in the tensor structure of the second term, $1/3$ in our case versus $1/2$ in GR. This difference indicates that our graviton contains addition degrees of freedom, on top of the usual two of the standard massless graviton. In particular, there is an additional scalar attraction. So is this theory ruled out?

Before answering this question, let me do a historic detour and discuss the analogous issue in massive gravity. In '71 van Dam, Veltman[8] and Zakharov [9] (vDVZ)

suggested that graviton mass, no matter how small, was excluded by solar system observations. This conclusion was based on the existence of the discontinuity of the linearized graviton propagator in massive versus massless theory. The propagator of the massive gravity, derived from the Pauli-Fierz action (the only ghost-free linearized action for a massive spin-2 particle) has the following form

$$D^{(m)}_{\mu\nu;\alpha\beta}(q) = \left(\frac{1}{2}\, \tilde{\eta}_{\mu\alpha}\tilde{\eta}_{\nu\beta} + \frac{1}{2}\, \tilde{\eta}_{\mu\beta}\tilde{\eta}_{\nu\alpha} - \frac{1}{3}\, \tilde{\eta}_{\mu\nu}\tilde{\eta}_{\alpha\beta} \right) \frac{1}{q^2 + m_g^2 - i\epsilon}\,, \tag{10}$$

where

$$\tilde{\eta}_{\mu\nu} = \eta_{\mu\nu} + \frac{q_\mu q_\nu}{m_g^2}\,. \tag{11}$$

Whereas, the massless graviton propagator is

$$D^{(0)}_{\mu\nu;\alpha\beta}(q) = \left(\frac{1}{2}\, \eta_{\mu\alpha}\eta_{\nu\beta} + \frac{1}{2}\, \eta_{\mu\beta}\eta_{\nu\alpha} - \frac{1}{2}\, \eta_{\mu\nu}\eta_{\alpha\beta} \right) \frac{1}{q^2 - i\epsilon}\,, \tag{12}$$

where only the momentum independent parts of the tensor structure is kept. By a gauge choice the momentum dependent structures can be taken to be zero. On the other hand, there is no such gauge freedom for the massive gravity.

Although the propagator of massive graviton contains terms singular in m_g, they vanish when convoluted with conserved sources, and thus, are irrelevant in one-particle exchange. Hence, if the one-graviton exchange were a good approximation, the vDVZ discontinuity indeed would rule out the massive gravity. Assumption of the one-graviton exchange dominance is therefore implicit in vDVZ analysis. However, in *massive* theory the validity of one-graviton approximation is questionable for solar system distances, as it was first argued by Vainshtein[10] shortly after vDVZ works.

In[11] the issue was reconsidered in the following light. Imagine that we wish to study the motion of a planet in the massive theory. We can try to find the metric in form of the G_N-exansion. But in order to trust this expansion, one has to be sure that the leading effect is indeed given by one-graviton exchange, that is by the contribution of order G_N. However, this is not always the case, because the additional polarizations of massive graviton have couplings singular in m_g which *do not necessarily* vanish beyond the linearized approximation. In fact one can directly check that the "subleading" diagrams proportional to G_N^2, at solar system distances are about 10^{32}-times the "leading" ones proportional to G_N! This invalidates vDVZ conclusion: although there is a discontinuity in the propagator, this discontinuity sheds no light on the phenomenological validity of the massive theory, since at solar system scales the linearized approximation for massive theory breaks down. Breakdown of the linearized gravitational approximation at the solar system distances may come as a surprise, however, we must remember that we are dealing with the modified theory that includes *additional* degrees of freedom (additional polarizations of massive graviton), not present in the massless theory. As shown in

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[11] the breakdown of perturbation theory is due to strong coupling of the longitudinal polarizations of the massive graviton, due to which a trilinear vertex exhibits $1/m_g^4$-type singularity (higher in $1/m_g$ singularities cancel). The same conclusion was reached in [12], using the Stuckelberg type formalism.

Although the breakdown of the perturbation theory invalidates the phenomenological objections based on vDVZ-discontinuity of the linearized graviton propagator, at this level the issue of the validity of the massive graviton theory remains inconclusive due to the lack of a fully generally-covariant ghost-free formulation of such a theory.[2] On the other hand, the model[1] is generally-covariant and is free of ghosts, and the argument can be taken through there. There too, the linearized propagator exhibits a vDVZ-type discontinuity in $r_c \to \infty$ limit, and there too the additional polarizations of graviton have couplings singular in $1/r_c$, invalidating perturbation theory at solar system scales[11]. However, the correctly resummed solutions can be found and are continuous.

The effective brane-to-brane graviton propagator can be written as

$$D_{\mu\nu;\alpha\beta}(q) = \left(\frac{1}{2}\tilde\eta_{\mu\alpha}\tilde\eta_{\nu\beta} + \frac{1}{2}\tilde\eta_{\mu\beta}\tilde\eta_{\nu\alpha} - \frac{1}{3}\tilde\eta_{\mu\nu}\tilde\eta_{\alpha\beta}\right)\frac{1}{q^2 + q/r_c - i\epsilon}, \qquad (13)$$

where

$$\tilde\eta_{\mu\nu} = \eta_{\mu\nu} + \frac{q_\mu q_\nu}{q/r_c}. \qquad (14)$$

and we see that it shares some important features with massive gravity in the sense that the additional polarizations are identical (one spin zero and two spin one), and the role of the mass is played by q/r_c term. This is not surprising since the 5D graviton from the point of view of the 4D theory looks like a resonance that can be spectrally expanded in continuum of massive spin-2 states (4), each of which has five polarizations. It can be directly checked that the tree-graviton vertex exhibits the following singularity [11]

$$\frac{q^3 r_c^2}{M_P} \qquad (15)$$

and hence the longitudinal polarizations become strongly coupled at the scale $q_s = (M_P/r_c^2)^{\frac{1}{3}}$. (This was also found in [15] in the language of [12]). What is the physical meaning of this scale? The breakdown of the linearized approximation and existence of the "strong coupling" signals that G_N is not any more a good expansion parameter, and series have to be re-summed. If the resummation is possible and the answer does not explicitly contain the "strong coupling" scale, then this scale is simply an artifact of the incorrect perturbative expansion in powers of G_N. The breakdown of the perturbative expansion (at least at the solar system scales) and the existence of the re-summation are the key points for avoiding the vDVZ conclusion[11].

At the tree-level there indeed is a complete resummation for different cases of interest[11, 16, 17]. For instance, Schwarzschild solution can be found in terms of

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[2]Nonlinear completions of Pauli-Fierz action also suffer from instabilities[13, 14].

$1/r_c$-expansions and exhibits a complete continuity. This solution has the following form [17]

$$\nu(r) \; = \; -\frac{r_g}{r} + \mathcal{O}\left(\frac{1}{r_c r}\sqrt{r_g r^3}\right) \,, \qquad \lambda(r) \; = \; \frac{r_g}{r} + \mathcal{O}\left(\frac{1}{r_c r}\sqrt{r_g r^3}\right) \,,$$

(16)

Here r_g is the gravitational radius of the body, and the functions $\nu(r)$ and $\lambda(r)$ parameterize the spherically-symmetric metric in the usual way $g_{00} = \mathrm{e}^{\nu(r)}$, $g_{rr} = \mathrm{e}^{\lambda(r)}$. As we see, any reference to the scale q_s has disappeared from the resummed solution. Which indicates that the "strong coupling" scale could indeed be an artifact of the perturbative expansion in terms of G_N.

We could ask the question about the effect of this strong coupling behavior on the loop corrections. It is obvious that since the perturbative expansion in G_N breaks down already at the tree-level, it will also break down in the loops. The question is whether a similar resummation should exist at the loop level, or the tree-level resummation is some sort of an accident? According to one possible believe [15], the loops should ruin resummation, but this is far from being obvious. So far, the only supporting argument for such a statement is that the perturbation theory breaks down in loops, which is obvious, as it breaks down already at the tree-level due to exactly the same non-linear interaction[11]. Unfortunately, the only way to compute loops in non-renormalizable theory is to know its ultraviolet (UV) completion. Simply cutting off the loop divergences at the presumed scale is illegitimate, and moreover adds nothing to our knowledge unless we have some independent information about the UV scale, and at least can do matching of scales (as e.g., in the chiral Lagrangians). UV-completion, such as embedding in string theory (see [18, 19]) potentially could answer this question, but at the level of the effective IR-theory the effect of the loop corrections is unclear. Phenomenologically, the most important question is not whether the perturbation theory breaks down (which is the fact), but whether the UV-completed theory (in which one can properly account for loops) affects the validity of the resummed solution obtained in the IR-theory. We shall discuss this issue in more details later on an explicit example that indicates that the UV-completion is not affecting the resummed solution.

Coming back to the solution (16), we notice that it does exhibit some interesting behavior. Let us focus on the form of the corrections to the ordinary Schwarzschild metric. This corrections are suppressed by powers of r/r_c as it should be the case by continuity. However they are also enhanced by powers of the ration r/r_g. This enhancement has a clear physical meaning. Indeed, as we have seen the continuity is restored by non-linearities that are important near the gravitating sources. This is where the effect of additional polarization diminishes. For the light sources, however, the non-linearities are unimportant and corrections due to additional polarizations become significant. As a result such deviations could be potentially observed near the light sources. To briefly summarize, there are the following important scales that govern the metric of gravitating spherically-symmetric bodies. These are r_g, r_c

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and a new scale

$$r_* = (r_c \sqrt{r_g})^{\frac{2}{3}} \tag{17}$$

Regimes of gravity alternate as follows. For $r \ll r_*$ gravity is essentially Einsteinian, with small corrections given by (16). For $r_* \ll r \ll r_c$ gravity is still $1/r$ but is of a scalar-tensor type. Finally for $r \gg r_c$ gravity becomes 5D $1/r^2$-type.

4   Meaning of the "Strong Coupling" Scales.

I wish now to discuss what are the physical scales of the theory. For example, when dealing with the perturbative expansion in series of G_N we have encountered a "strong coupling" type behavior at the scale q_s. What is the meaning of this scale, and what are its effects on observations? As mentioned above, one possibility is that this scale is simply an artifact of the perturbative expansion in G_N, and should disappear when the perturbative series are correctly resummed. This possibility is strongly supported by the existence of the fully-resummed tree-level solutions. If the loop expansion would admit similar resummation, then the scale q_s would be unphysical.

There of course remains an option that the tree-level resummation is some sort of an accident, and there is no analog effect at the loop-level. In such a case the scale q_s may be physical, meaning that some physical degrees of freedom, not accounted in the effective low energy theory, have to be introduced around this scale. The question we want to ask now is how strongly these new degrees of freedom modify the resummed tree-level solutions of the effective theory above the scale q_s. In the other words, how much the solutions obtained in low energy effective theory differ from the analogous solutions obtained in UV-completed theory above the scales q_s? If the tree-level resummation is an accident, then the usual expectation is that the two solutions should differ dramatically above the scale q_s. However, the situation is more subtle. We wish to argue that in many cases the form of the tree-level solution is unaffected by the UV-completion, including loop corrections.

We shall now discuss an explicit toy model with the above property. It is a model with a non-Abelian Goldstone field, whose effective low energy description shares many similarities with the gravity case, including a strong-coupling behavior above the certain scale. However, the explicit UV-completion of this model demonstrates that classical solutions on the brane are unaffected by the UV-completing physics even above the naive "strong coupling" scale.

Consider the following low energy action

$$S = M_*^3 \int d^5 x \, \partial^A u^a \partial_A u^a + M^2 \int d^4 x \, \partial^\mu u^a \partial_\mu u^a \tag{18}$$

Where A is the five-dimensional index, and M and M_* are some mass parameters. Below we shall be interested in the limit $M_* \ll M$. u^a, $a = 1, ... 4$ is an $O(4)$-vector obtained by an arbitrary coordinate-dependent $O(4)$-rotation of the

unit vector $(1, 0, 0, 0)$. This rotation can be parameterized by the three Goldstone angles that we shall call $\phi(x), \theta(x)$ and $\chi(x)$. The vector u then can be written in the following form

$$u = (c_\phi s_\theta s_\chi, \ s_\phi s_\theta s_\chi, \ c_\theta s_\chi, \ c_\chi) \tag{19}$$

where c and s stand for Cos and Sin functions of the respective angles. Theory is obviously invariant under a global $O(4)$-rotation under which the Goldstone fields transform non-linearly, whereas the vacuum is only $O(3)$-invariant. The model (18) is a close analog of the model of infrared-modified gravity given by the action (2). Goldstone fields play the role analogous to graviton. Just like gravitons Goldstones are also universally coupled, and their non-linear interactions also exhibit strong coupling behavior. Because of the interplay of the two kinetic terms, the above model also exhibits the shielding phenomenon, due to which the linearized interactions on the brane interpolate between 4D and 5D regimes at the crossover scale $r_c = M^2/M_*^3$.

The effective theory has at least one "obvious" strong coupling scale, M_*. Indeed, the interactions of the 5D Goldstone fields are cut-off by this scale. So naively any classical solution on the brane, losses validity above the energy M_*. Let us see if this is indeed the case. Consider one such solution, which occurs for a spherically-symmetric brane of radius R. To build it up, let us first restrict our attention to the 4D world-volume of the brane, ignoring the fifth dimension. The world-volume theory is a theory on a three-sphere, with the metric $ds^2 = dt^2 - R^2[d\hat{\chi}^2 + s_{\hat{\chi}}^2(d\hat{\theta}^2 + s_{\hat{\theta}}^2 d\hat{\phi}^2)]$, where $\hat{\chi}$, $\hat{\theta}$, $\hat{\phi}$ are the usual angular variables. Since the vacuum manifold is also a three-sphere, on such a space there is a topologically non-trivial solution, the "texture" [20], which is obtained by mapping $S_3 \to S_3$. Texture is obtained from (19) by assuming the following coordinate dependence of the Goldstone fields $\chi = \hat{\chi}, \theta = \hat{\theta}, \phi = \hat{\phi}$. The energy density of the texture comes purely from the Goldstone gradient energy and scales as

$$E \sim M^2/R^2 \tag{20}$$

Hence, in truly four-dimensional theory given by the second term in (18), the solution would be valid all the way to the curvature radius $R \sim M^{-1}$. We may expect that the extension of the Goldstone model into the 5D space with lower cut-off M_*, should jeopardize this validity above the scale M_*. Indeed, let us first note that the solution can be easily embedded in 5D space, if $\hat{\chi}$, $\hat{\theta}$, $\hat{\phi}$ are identified with spherical coordinates in 5D space and we place the brane at $\rho = R$ (where ρ is the radial coordinate in 5D). The resulting configuration has the form of the hedgehog configuration of the vector u

$$u_{text} = (c_{\hat{\phi}} s_{\hat{\theta}} s_{\hat{\chi}}, \ s_{\hat{\phi}} s_{\hat{\theta}} s_{\hat{\chi}}, \ c_{\hat{\theta}} s_{\hat{\chi}}, \ c_{\hat{\chi}}) \tag{21}$$

In an effective theory (18) this solution breaks down at distances $\rho \ll M_*$, where the five-dimensional angular gradients blow-up. Thus, if the brane has a radius $R \gg 1/M_*$ the 4D texture solution is a good approximation on the brane, but

should break down for the radius $1/M \ll R \ll 1/M_*$. Let us now show that this naive expectation is not supported in the UV-completed theory. Although, the solution gets modified in the bulk, the world-volume configuration remains a valid solution all the way to the scale of UV-completed theory which can be arbitrarily higher than any strong-coupling scale of the effective IR theory.

The obvious UV completion of the above effective theory is given by the Goldstone model with a spontaneously broken $O(4)$-symmetry by the Higgs field Φ^a transforming as a four-component vector. The action for Φ we choose in the following way

$$S = \int d^5x \left(\partial_A \Phi^a \partial^A \Phi^a - \frac{1}{2M}(\Phi^a \Phi^a - M_*^3)^2 \right) - \int d^4x \frac{1}{2M^2} \left(\Phi^a \Phi^a - M^3 \right)^2 \tag{22}$$

The cutoff of the full UV-completed theory is M. This theory describes a spontaneously broken $O(4)$-symmetry, but the scales of breaking on the brane and in the bulk are very different. Scale of breaking in the five-dimensional bulk is M_*, whereas, as we shall demonstrate in a moment, the scale of breaking in the brane world-volume is $\sim M$. We expect that there will be all possible high-dimensional operators suppressed by powers of the scale M generated from the quantum loops. These, however, cannot affect any of our conclusions as long as we keep the hierarchy $M_* \ll M$. This addresses the issue of the loop corrections in UV-completed theory. The difference from the previously discussed sigma model is that the absolute value of Φ can fluctuate, or in the other words the Goldstones are supplemented by the Higgs degree of freedom

$$\Phi = f(x)u(x) \tag{23}$$

The mass of the Higgs excitation in the bulk is $m_{Higgs}^2 \sim M_*^3/M$. Hence, the effective low energy description in terms of the Goldstone degrees of freedom in the bulk breaks down at the scale M_{Higgs}. Despite this fact the non-linear solution of the full UV-completed theory on the brane is well described by the solution (21) of the sigma model all the way to the scale M. As a result, for the brane world-volume observers the classical solutions of the Goldstone theory remain valid up to the energies of order M. Before convincing ourselves in this validity, let us show that in the lowest energy state Φ indeed develops a large expectation value on the brane. It is easiest to prove this in the limit $M_* \to 0$, in which case Φ assumes a zero expectation value in the bulk. The fact that Φ wants to condense on the brane can be seen by examining the linearized equation for small perturbations about the $\Phi = 0$ solution in the brane background, which has the following form (due to $O(4)$-symmetry it is enough to consider a single component of Φ)

$$\left(\nabla^2 - \partial_y^2 - \delta(y)M \right) \Phi = 0 \tag{24}$$

It is obvious that this equation has a normalizable exponentially growing (imaginary frequency) tachyonic eigen-mode, localized on the brane

$$\Phi = e^{\frac{1}{2}Mt} e^{-\frac{1}{2}|y|M} \tag{25}$$

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This instability signals that Φ condenses on the brane and develops a large expectation value $\sim M$, as both the tachyonic mass as well as stabilizing potential are set by a single mass scale M. Hence, the expectation value on the brane is $\sim M$ and in the bulk is M_*. Then the low energy action for the Goldstone fields is similar to (18). Let us see what is the effect of this UV-completion on the texture solution on the brane at energies $\gg M_*$.

For the spherically symmetric brane of radius R, the texture solution is embedded in 5D in form of the hedgehog configuration of the scalar field, which in the five dimensional spherical coordinates can be written as

$$\Phi = f(\rho) \, u_{text} \tag{26}$$

where the angular dependence of u_{text} is as given in (21). The difference from the analogous solution in the Goldstone model, is that the absolute value of the vector Φ^a (given by function $(f(\rho))$) can now respond to the phase gradients, and vanish in the regions in which the phase-gradients become more costly than the Higgs energy. The phase gradient energy density is of order

$$E_{grad} \sim \frac{f(\rho)^2}{\rho^2} \tag{27}$$

This has to be compared with bulk and brane Higgs energy densities. For distances $\rho \ll \sqrt{M/M_*^3}$ the bulk Higgs energy density becomes less costly than the gradient energy, and $O(4)$ symmetry gets restored. Hence, in the bulk $f(\rho)$ vanishes inside a four-dimensional sphere of radius $\rho \ll M_{Higgs}^{-1}$. This scale, however, cannot affect the validity of the non-linear brane world-volume solution. Even for the branes with the size $R << 1/M_*$ this solution remains intact. This is obvious, since on the world-volume the Higgs energy is more costly than the gradient energy all the way to the scales $\rho \sim 1/M$. Hence, the expectation value of Φ on the brane remains non-zero as long as the brane radius $R \gg 1/M$. Thus, even for the brane sizes $1/M \ll R \ll 1/M_*$ the solution of the IR-theory stays in tact.

To interpolate between the different regimes, we can first take $R \gg 1/M_{Higgs}$ and gradually decrease it to the values $R \ll 1/M_*$. As long as $R \gg 1/M_{Higgs}$ the 5D-gradients of the Goldstone phases in the immediate neighborhood of the brane are small compared to the bulk Higgs energy . Then the 5D Higgs potential wins and the Higgs field stays in the vacuum both on the brane as well as in the bulk. Hence solution (26) of the UV-completed theory coincides with the solution of IR-theory both on the brane as well as in the bulk.

When the brane shrinks to the radius $1/M \ll R \ll 1/M_{Higgs}$, the 5D-Higgs potential becomes sub-dominant to the 5D-gradients of the phase, and can no longer support the non-zero expectation value of Φ outside the brane. In the other words, the brane enters inside the core of the hedgehog solution, where symmetry is restored. However, a four-dimensional observer on the brane continues to see the spontaneously-broken $O(4)$-symmetry at scale M, because the phase-gradients are

much less costly than the world-volume Higgs energy. So texture solution on the brane is unaffected by the UV-regulating physics. For example, to unwind the texture on the world-volume, we have to make Φ vanish there, which is more costly then the gradient energy. Hence Φ stays in the vacuum inside the brane, and texture solution stays in tact. For the four-dimensional observer on the brane the situation looks identical to what it would look in 4D space with spontaneously broken $O(4)$ symmetry at scale M. Hence, there is no reference to the scales $\sqrt{M/M_*^3}$ or M_* as far as the solution on the brane is concerned, even above the eneregies of order M_*!

In the other words the strongly coupled bulk physics got "shielded" from the brane observer by the huge brane-localized kinetic term. Although the above example is not gravity, it shows the generic point that UV-completions act "softly" on the brane-solutions in shielded theories. Hence, in this class of theories, IR-solutions can still be used for confronting observations.

5   Anomalous Perihelion Precession.

The considered theory predicts the slight modification of the gravitational potential of a massive body at the observed distances according to (16). This modification can be observable in the experiments that are sensitive to anomalous perihelion precession of planets[21]. Following the analysis of [21], let ϵ be the fractional change of the gravitational potential

$$\epsilon \equiv \frac{\delta \Psi}{\Psi}, \tag{28}$$

where $\Psi = -GM/r$ is the Newtonian potential. The anomalous perihelion precession (the perihelion advance per orbit due to gravity modification) is

$$\delta \phi = \pi r (r^2 (r^{-1} \epsilon)')', \tag{29}$$

where $' \equiv d/dr$.

Let us apply this to the model of [1]. In this theory, [17]

$$\epsilon = -\sqrt{2} r_c^{-1} r_g^{-1/2} r^{3/2}. \tag{30}$$

The numerical coefficient deserves some clarification. The above coefficient was derived in [17] on Minkowski background. However, non-linearities created by cosmological expansion can further correct the coefficient. One would expect these corrections to scale as powers of $r_c H$, where H is the observed value of the Hubble parameter. On the accelerated branch, as it's obvious from (7), $H \sim 1/r_c$ and thus, one would expect the corrections to be of order one. We will restrict ourselves to order of magnitude estimate, but the sign may be important if the effect is found, since according to[22] it could give information about the cosmological branch. We get

$$\delta \phi = (3\pi/4)\epsilon. \tag{31}$$

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Numerically, the gravitational radius of the Earth is $r_g = 0.886$cm, the Earth-Moon distance is $r = 3.84 \times 10^{10}$cm, the gravity modification parameter that gives the observed acceleration without dark energy $r_c = 6$ Gpc. We get the theoretical precession

$$\delta\phi = 1.4 \times 10^{-12}. \tag{32}$$

This is to be compared to the accuracy of the precession measurement by the lunar laser ranging. Today the accuracy is $\sigma_\phi = 4 \times 10^{-12}$ and no anomalous precession is detected at this accuracy, in the near future a tenfold improvement of the accuracy is expected[23]. Then a detection of gravity modification predicted by [1] will be possible.

Although we have focused on a particular model of large-distance gravity modification, we expect that the property that the corrections to the ordinary gravity near the weak sources should set in at relatively shorter distances should be shared by the whole class of such theories. The reason is that, as argued above, the theories that modify gravity in far IR propagate additional polarizations, and hence, at the linearized level give the results different from GR (or else they have to contain ghosts, and are not viable). So in order to be compatible with observations, in these theories the naive perturbative expansions in G_N must break down at least at the solar system distances, and strong coupling of additional polarizations precisely accomplishes this role[11]. The correctly re-summed solutions should exhibit continuity in $1/r_c$. From this reasoning it follows that in any theory that passes the solar system tests the resummed solution near the gravitating sources must have the property that the relative deviations from the GR are enhanced by powers of r/r_g. Indeed, for small r_g (the weak sources) the linearized gravity, which is by order one different from the Einsteinian counterpart for the same source, must become a better approximation at relatively shorter distances than for the heavy sources. Hence the continuity demands that at a fixed distance r the corrections should increase with decreasing r_g. Thus, the fractional corrections to the Einsteinian gravitational potential are expected to be of the form

$$\epsilon \sim r_c^{-1-\gamma} r_g^{-1/2} r^{3/2+\gamma}, \tag{33}$$

where γ is model-dependent. Notice that the fact that the cosmological evolution of the Universe should be dramatically modified when the horizon becomes comparable to r_c, directly follows from the above expression. Indeed, treating the Universe as the source with the gravitational radius of $r_g \sim r_c$, the fractional correction ϵ becomes order one. The anomalous lunar perihelion precession is

$$\delta\phi \sim r_c^{-1-\gamma} r_g^{-1/2} r^{3/2+\gamma}. \tag{34}$$

Observations of accuracy σ_ϕ can therefore test gravity theories with

$$r_c < r \left(\frac{r}{\sigma_\phi^2 r_g} \right)^{\frac{1}{2(1+\gamma)}}. \tag{35}$$

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One can speculate that in the absence of additional scales the gravity theories that produce self-acceleration, without vacuum energy should have $r_c \sim$few Gpc. Then the lunar precession accuracy of $\sigma_\phi \sim 10^{-12}$ will tests the $\gamma = 0$ theory[1].

6 Discussion and Outlook.

Possibility of large-distance modification of gravity is a fundamental question, motivated by the Dark Energy[4, 7, 24, 28] and the Cosmological Constant[25, 26, 27] problems. In this lecture we have addressed the issue of viability and observable impact of such a modification. Although for the illustrative purposes we have focused mostly on a concrete model (2), our conclusions are very general and apply to large class of theories. In particular, this class should include any generally-covariant theory in which the graviton propagator admits a sensible spectral representation in terms of positive norm mass- and spin-eigenstates.

The following generic features emerged. IR-modified gravity theories propagate additional degrees of freedom (additional polarizations of graviton). Due to these addition states, at the the linearized level the predictions of modified theories are by order one different from the predictions of GR. Hence, the only way the new theories could be compatible with the experiment is if the naive perturbation theory in terms of G_N-expansion breaks down, at least at solar system distances. It is interesting that in the discussed model this is precisely what happens, due to the strong coupling of the additional polarizations of the graviton[11]. Appearance of the strong coupling in the naive perturbative expansion is the only reason why these theories are not immediately ruled out. At the classical level there is a full resummation and non-linear solutions exhibit continuity in $1/r_c$. The naive strong coupling scale disappears from these solutions. Of course, there is a question of loops, which is impossible to answer without knowing the UV-completion. Doing loop expansion in terms of G_N and simply cutting off the loop divergences at the suspected scale, knowing that the perturbation theory in the very same expansion parameter breaks down already at the tree-level, adds nothing to our understanding of the situation, unless we have some information about the UV-completion.

However, even without knowing the precise form of the UV-completion, one could envisage the two options. One not unlikely possibility is that the tree-level resummation is not an accident, and indicates that an analogous resummation should take place in the loops. In such a case the scale q_s should be disregarded as the artifact of the incorrect perturbation theory.

It is certainly possible that the resummation is an accident, and does not go beyond the three-level. Then the perturbative strong coupling probably indicates that there are new degrees of freedom at the scale q_s and these should be integrated in. In such a case the theory above q_s will be described by the UV-completed theory that will include these new degrees of freedom. The crucial question then is how different are the fully-resummed non-linear solutions of IR-theory, which exhibit no

pathology above the scale q_s, in comparison with the analogous solutions of the UV-completed theory? Although, one may naively expect that the difference between the two must become dramatic above the scale q_s, the suggestive toy examples exhibit the opposite behavior. The solution on the brane is essentially unaffected by the UV-completion. We suspect that this may be a property of the theories in which four-dimensional behavior on the brane is obtained by shielding. This connection will be discussed elsewhere.

Due to new degrees of freedom, the theories with IR-modified gravity are very constrained and can be potentially tested not only by the precision cosmology, but also by shorter scale gravitational measurements, and in some models (that predict an especially low fundamental scale of gravitational interaction) may be probed by the upcoming collider experiments [2].

Note Added

Since this talk was given there have been some interesting developments, and for brevity, I shall only comment on the ones that are relevant for the issue of the "strong coupling" discussed here. Modifying the model [1], Gabadadze and Shifman [29] (see also [30]) have constructed an example that even in the straightforward perturbative expansion remains weakly coupled till the scale M_*. This is an interesting existence proof, perhaps complementary to the approach outlined in this talk, according to which the breakdown of the perturbative G_N-expansion is the way for making IR-modified gravity (with additional polarizations) compatible with observations. Luckily this is accomplished by the singular in $1/r_c$ "strong coupling" of extra polarizations, as discussed in [11]. The key to the issue is in correct resummation which eliminates the dependence of the physical solutions on the strong coupling scale.

Acknowledgments

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