

Nonlocal action for long-distance modifications of gravity theory

A.O.Barvinsky[†]

Theory Department, Lebedev Physics Institute, Leninsky Prospect 53, Moscow 117924, Russia

Abstract

We construct the covariant nonlocal action for recently suggested long-distance modifications of gravity theory motivated by the cosmological constant and cosmological acceleration problems. This construction is based on the special nonlocal form of the Einstein-Hilbert action explicitly revealing the fact that this action within the covariant curvature expansion begins with curvature-squared terms. Nonlocal form factors in the action of both quantum and brane-induced nature are briefly discussed. In particular, it is emphasized that for certain class of quantum initial value problems nonlocal nature of the Euclidean action does not contradict the causality of effective equations of motion.

[†]e-mail: barvin@td.lpi.ru

1. Introduction

The purpose of this paper is to suggest the class of nonlocal actions for covariantly consistent infrared modifications of Einstein theory discussed in [1]. The modified equations of motion were suggested to have the form of Einstein equations

$$M_P^2 \left(1 + \mathcal{F}(L^2 \square)\right) \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right) = \frac{1}{2} T_{\mu\nu} \quad (1.1)$$

with "nonlocal" inverse gravitational constant or Planck mass, $M_P^2(\square) = M_P^2 (1 + \mathcal{F}(L^2 \square))$, being some function of the dimensionless combination of the covariant d'Alembertian $\square = g^{\alpha\beta} \nabla_\alpha \nabla_\beta$ and the additional scale L – the length at which infrared modification becomes important, $1/\sqrt{-\square} \sim L$. If the function of this dimensionless combination $z = L^2 \square$ satisfies the conditions

$$\begin{aligned} \mathcal{F}(z) &\rightarrow 0, & z &\gg 1 \\ \mathcal{F}(z) &\rightarrow \mathcal{F}(0) \gg 1, & z &\rightarrow 0, \end{aligned} \quad (1.2)$$

then the long-distance modification is inessential for processes varying in spacetime faster than $1/L$ and is large for slower phenomena at wavelenghtes $\sim L$ and larger. This opens the prospects for resolving the cosmological constant problem, provided one identifies the scale L with the horizon size of the present Universe $L \sim 1/H_0 \sim 10^{28} \text{cm}$. Indeed, equations (1.2) then interpolate between the Planck scale of the gravitational coupling constant $G_P = 16\pi/M_P^2$ for local matter sources of size $\ll L$ and the long distance gravitational constant $G_{LD} = 16\pi/M_P^2(1 + \mathcal{F}(0)) \ll G_P$ with which the sources nearly homogeneous at the horizon scale L are gravitating. Therefore, the vacuum energy \mathcal{E} , $T_{\mu\nu} = \mathcal{E}g_{\mu\nu}$, of TeV or even Planckian scale (necessarily arising in all conceivable models with spontaneously broken SUSY or in quantum gravity) will not generate a catastrophically big spacetime curvature incompatible with the tiny observable H_0^2 . This mechanism is drastically different from the old suggestions of supersymmetric cancellation of \mathcal{E} [2], because it relies on the fact that the nearly homogeneous vacuum energy gravitates very little, rather than it is itself very small¹. It will generate the curvature $H^2 \sim G_{LD}\mathcal{E} \sim G_P \mathcal{E}/\mathcal{F}(0)$ which can be very small due to large $\mathcal{F}(0)$.

Various aspects of this idea have been discussed in much detail in [1]. One formal difficulty with this construction was particularly emphasized by the authors of [1]. Point is that for any nontrivial form factor $\mathcal{F}(L^2\Box)$ the left hand side of (1.1) does not satisfy the Bianchi identity and, therefore, cannot be generated by generally covariant action. The equations (1.1) are generally covariant, but cannot be represented as a metric variational derivative of the diffeomorphism invariant action. Obviously, this makes the situation unsatisfactory because of a missing off-shell extension of the theory, problems with its quantization, etc.

In this paper we suggest to circumvent this problem by the following simple observation. Point is that the infrared regime, which is crucial for the resolution of the cosmological constant problem, implies not only the long-wavelength but also the *weak field* approximation. This means that the equation (1.1) is literally valid only as a first term of the perturbation expansion in powers of the curvature. Therefore, the left hand side of (1.1) should be modified by higher than linear terms in the curvature, and the modified nonlocal action $S_{NL}[g]$ should be found from the variational equation

$$\frac{\delta S_{NL}[g]}{\delta g_{\mu\nu}(x)} = M_P^2 g^{1/2} (1 + \mathcal{F}(L^2\Box)) \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) + \text{O}[R_{\mu\nu}^2]. \quad (1.3)$$

Flexibility in higher orders of the curvature allows one to guarantee the integrability of this equation and to construct the nonlocal action as a generally covariant (but

¹The idea of the scale dependent Newton's constant was also suggested in [3] within curved-brane models, though it was not explicitly formulated in terms of a nonlocal form factor.

nonlocal) curvature expansion. Here we explicitly present this construction along the lines of covariant curvature expansion developed in [4, 5, 6] a number of years ago. As a starting point we consider a special nonlocal form of the Einstein-Hilbert action revealing its basic property – the absence of a linear in metric perturbation part (on flat-space background), which is apparently the classical analogue of a tadpole elimination technique in non-SUSY string models [7]. Then we introduce a needed long-distance modification by a simple replacement of the nonlocal form factor in the curvature-squared term of the obtained action. The paper is accomplished by a discussion of the nature of nonlocalities in quantum-gravitational and brane-induced models of [8, 9, 10]. In particular, the fact that curvature expansion for the action begins with the quadratic order is revisited from the viewpoint of the running gravitational coupling constant. Finally, the issue of acausality of nonlocal effective equations, raised in [1], is reconsidered and a possible generalization to asymptotically deSitter spacetimes is briefly discussed.

2. Nonlocal form of the Einstein action

For simplicity, we start with the Euclidean (positive-signature) asymptotically-flat spacetime in d dimensions. The action of Einstein theory

$$S_E[g] = -M_P^2 \int dx g^{1/2} R(g) - 2M_P^2 \int_{\infty} d^{d-1}\sigma (g^{(d-1)})^{1/2} (K - K_0) \quad (2.1)$$

includes the bulk integral of the d -dimensional scalar curvature and the surface integral over spacetime infinity, $|x| \rightarrow \infty$, with induced metric g^{d-1} . The latter is usually called the Gibbons-Hawking action which in the covariant form contains the trace of the extrinsic curvature of the boundary K (with the subtraction of the flat space background K_0). This surface term guarantees the consistency of the variational problem for this action which yields as a metric variational derivative the Einstein tensor

$$\frac{\delta S_E[g]}{\delta g_{\mu\nu}(x)} = M_P^2 g^{1/2} \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right). \quad (2.2)$$

The action is local and manifestly covariant, but it contains together with the spacetime metric auxiliary structures – as a part of boundary conditions it involves at spacetime infinity the vector field normal to the boundary and the corresponding extrinsic curvature. As we will now see these structures can be identically excluded from the action without losing covariance, but by the price of locality – the local action will be transformed to the manifestly nonlocal form which will serve as a hint for constructing *covariant* long-distance modifications.

Another property of the action (2.1) is that it is explicitly linear in the curvature. However, this linearity is in essence misleading, because the variational derivative (2.2) is also linear in curvature and, therefore, it is at least linear in metric perturbation on flat-space background $R_{\mu\nu} \sim h_{\mu\nu}$. Thus the flat-space perturbation theory for the Einstein action should start with the quadratic order, $O[h_{\mu\nu}^2] \sim O[R_{\mu\nu}^2]$. This is a well known fact from the theory of free massless spin-2 field. Our goal is to make this $h_{\mu\nu}$ -expansion manifestly covariant, that is to convert it to the covariant (but generally nonlocal) expansion in powers of the curvature. A systematic way to do this is to use the technique of covariant perturbation theory of [4, 5, 6]. This technique begins with the derivation of the expression for the metric perturbation in terms of the curvature and in our context looks as follows.

Expand the Ricci curvature in metric perturbations on flat-space background

$$R_{\mu\nu} = -\frac{1}{2}\square h_{\mu\nu} + \frac{1}{2}\nabla_\mu\left(\nabla^\lambda h_{\nu\lambda} - \frac{1}{2}\nabla_\nu h\right) + \frac{1}{2}\nabla_\nu\left(\nabla^\lambda h_{\mu\lambda} - \frac{1}{2}\nabla_\mu h\right) + O[h_{\mu\nu}^2] \quad (2.3)$$

and solve it by iterations as a nonlocal expansion in powers of the curvature. This expansion starts with the following terms

$$h_{\mu\nu} = -\frac{2}{\square}R_{\mu\nu} + \nabla_\mu f_\nu + \nabla_\nu f_\mu + O[R_{\mu\nu}^2]. \quad (2.4)$$

Here $1/\square$ acting on $R_{\mu\nu}$ denotes the action of the Green's function $G_{\mu\nu}^{\alpha\beta}(x, y)$ of the *covariant metric-dependent* d'Alembertian $\square\delta_{\mu\nu}^{\alpha\beta} \equiv g^{\lambda\sigma}\nabla_\lambda\nabla_\sigma\delta_{\mu\nu}^{\alpha\beta}$ on the space of symmetric second-rank tensors with natural zero boundary conditions at infinity

$$\frac{1}{\square}R_{\mu\nu}(x) \equiv \frac{\delta_{\mu\nu}^{\alpha\beta}}{\square}R_{\alpha\beta}(x) = \int dy G_{\mu\nu}^{\alpha\beta}(x, y) R_{\alpha\beta}(y), \quad (2.5)$$

$\square_x G_{\mu\nu}^{\alpha\beta}(x, y) = \delta_{\mu\nu}^{\alpha\beta}\delta(x, y)$, $G_{\mu\nu}^{\alpha\beta}(x, y)|_{|x|\rightarrow\infty} = 0$. In what follows we will not specify the tensor structure of the Green's functions of \square implicitly assuming that it is always determined by the nature of the quantity acted upon by $1/\square$.

The term $\nabla_\mu f_\nu + \nabla_\nu f_\mu$ in (2.4) reflects the gauge ambiguity in the solution of (2.3) for $h_{\mu\nu}$ (originating from the harmonic-gauge terms in the right-hand side of (2.3)), but its explicit form is not important for our purposes here².

²The only important property of this term is that this is a gauge transformation with some gauge parameter $f_\mu \sim \nabla^\nu h_{\mu\nu} - \nabla_\mu h/2 + O[h_{\mu\nu}^2]$. Explicit gauge fixing procedure for the equation (2.3) becomes important in higher orders of curvature expansion and it is presented in much detail in [4, 5, 6].

Now restrict ourselves with the approximation quadratic in $R_{\mu\nu}$ (or equivalently, $h_{\mu\nu}$) and integrate the variational equation (2.2) for $S_E[g]$. Since the variational derivative is at least linear in $h_{\mu\nu}$, $\delta S_E/\delta g_{\mu\nu} \sim h_{\alpha\beta}$, the quadratic part of the action in view of this equation is given by the integral

$$S_E[g] = \frac{1}{2} \int dx h_{\mu\nu}(x) \frac{\delta S_E[g]}{\delta g_{\mu\nu}(x)} + \mathcal{O}[R_{\mu\nu}^3]. \quad (2.6)$$

Substituting (2.2) and (2.4) and integrating by parts one finds that the contribution of the gauge parameters f_μ vanishes in view of the Bianchi identity for the Einstein tensor, and the final result reads

$$S_E[g] = M_P^2 \int dx g^{1/2} \left\{ -\left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R\right) \frac{1}{\square} R_{\mu\nu} + \mathcal{O}[R_{\mu\nu}^3] \right\}. \quad (2.7)$$

This is the covariant *nonlocal* form of the *local* Einstein action which was originally observed in our previous papers on braneworld scenarios with two repulsive branes [11, 12]. This nonlocal incarnation of (2.1) explicitly features: i) the absence of linear in curvature term and ii) the absence of auxiliary structures associated with spacetime infinity. Before we go over to the construction of long-distance modifications of the theory, let us briefly dwell on higher-order curvature terms. This, in particular, will clarify the role played by the Gibbons-Hawking action in the subtraction of the linear term.

In asymptotically-flat (Euclidean) spacetime with the asymptotic behavior of the metric

$$g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}, \quad h_{\mu\nu} = \mathcal{O}\left(\frac{1}{|x|^{d-2}}\right), \quad |x| \rightarrow \infty, \quad (2.8)$$

the noncovariant form of the Gibbons-Hawking term in Cartesian coordinates reads as

$$\begin{aligned} S_{\text{GH}}[g] &\equiv -2 M_P^2 \int_\infty d^{d-1} \sigma \left(g^{(d-1)}\right)^{1/2} (K - K_0) \\ &= M_P^2 \int_{|x| \rightarrow \infty} d\sigma^\mu (\partial^\nu h_{\mu\nu} - \partial_\mu h). \end{aligned} \quad (2.9)$$

This surface integral can be transformed to the bulk integral of the integrand $\partial^\mu (\partial^\nu h_{\mu\nu} - \partial_\mu h)$ – the linear in $h_{\mu\nu}$ part of the scalar curvature. From the viewpoint of the metric in the interior of spacetime this is a topological invariant depending only on the asymptotic behavior $g_{\mu\nu}^\infty = \delta_{\mu\nu} + h_{\mu\nu}(x)|_{|x| \rightarrow \infty}$. Similarly to the above procedure this integral can be covariantly expanded in powers of the curvature. Up to cubic terms

inclusive this expansion reads³

$$\begin{aligned}
\int_{\infty} d\sigma^{\mu} (\partial^{\nu} h_{\mu\nu} - \partial_{\mu} h) = & \int dx g^{1/2} \left\{ R - \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) \frac{1}{\square} R_{\mu\nu} \right. \\
& + \frac{1}{2} R \left(\frac{1}{\square} R^{\mu\nu} \right) \frac{1}{\square} R_{\mu\nu} - R^{\mu\nu} \left(\frac{1}{\square} R_{\mu\nu} \right) \frac{1}{\square} R \\
& + \left(\frac{1}{\square} R^{\alpha\beta} \right) \left(\nabla_{\alpha} \frac{1}{\square} R \right) \nabla_{\beta} \frac{1}{\square} R \\
& - 2 \left(\nabla^{\mu} \frac{1}{\square} R^{\nu\alpha} \right) \left(\nabla_{\nu} \frac{1}{\square} R_{\mu\alpha} \right) \frac{1}{\square} R \\
& \left. - 2 \left(\frac{1}{\square} R^{\mu\nu} \right) \left(\nabla_{\mu} \frac{1}{\square} R^{\alpha\beta} \right) \nabla_{\nu} \frac{1}{\square} R_{\alpha\beta} + O[R_{\mu\nu}^4] \right\}. \quad (2.10)
\end{aligned}$$

As we see, when substituting to (2.1) its linear term cancels the Ricci scalar part, the quadratic terms reproduce those of (2.7) and the cubic terms recover $O[R_{\mu\nu}^3]$. Obviously, this type of expansion can be extended to arbitrary order in curvature.

3. Long-distance modification of the Einstein action

Long distance modification of the Einstein action that would generate (1.3) as the left-hand side of the gravitational equations of motion now can be simply obtained from the nonlocal form of the Einstein action (2.7). It is just enough to make the following replacement in the quadratic part of (2.7)

$$\frac{1}{\square} \rightarrow \frac{1 + \mathcal{F}(L^2 \square)}{\square}. \quad (3.1)$$

Indeed, the subsequent variation of the Ricci tensor, $\delta_g R_{\mu\nu} = -\frac{1}{2} \square \delta g_{\mu\nu} + \nabla_{\mu} f_{\nu} + \nabla_{\nu} f_{\mu}$, in

$$\begin{aligned}
\delta_g \int dx g^{1/2} \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) \frac{1 + \mathcal{F}(L^2 \square)}{\square} R_{\mu\nu} = \\
2 \int dx g^{1/2} \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) \frac{1 + \mathcal{F}(L^2 \square)}{\square} \delta_g R_{\mu\nu} + O[R_{\mu\nu}^2] \quad (3.2)
\end{aligned}$$

and integration by parts "cancel" the denominator of (3.1), whereas the contribution of gauge parameters f_{μ} vanishes, as above, in view of the Bianchi identity. All commutators of covariant derivatives with the \square in the form factor (3.1) give rise to the

³Validity of this result can be checked either by the direct $h_{\mu\nu}$ -expansion of the right-hand side or by systematically expanding $h_{\mu\nu}$ on the left hand side as covariant series in the curvature, starting with (2.4) [4, 5, 6].

curvature-squared order which is beyond our control. This recovers the Einstein tensor term of (1.3) with the needed "nonlocal" Planckian mass $M_P^2(1 + \mathcal{F}(L^2\Box))$.

The result of the replacement (3.1) can be rewritten so that the contribution of 1 in the numerator of the new form factor is again represented in the usual local form of the Einstein action (2.1). Then, the long-distance modification takes the form of the additional nonlocal term

$$S_{NL}[g_{\mu\nu}] = S_E[g_{\mu\nu}] - M_P^2 \int dx g^{1/2} \left\{ \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) \frac{\mathcal{F}(L^2\Box)}{\Box} R_{\mu\nu} + \mathcal{O}[R_{\mu\nu}^3] \right\}. \quad (3.3)$$

This term is not unique though, because it is defined by a given form factor $\mathcal{F}(L^2\Box)$ only in quadratic order, while we do not have good principles to fix its higher-order terms thus far.

This action is manifestly generally covariant. Therefore, its variational derivative (the left hand side of the modified Einstein equations) exactly satisfies the Bianchi identity,

$$\nabla_\mu \frac{\delta S_{NL}[g_{\mu\nu}]}{\delta g_{\mu\nu}(x)} = -M_P^2 g^{1/2} \nabla_\mu \left[\left(1 + \mathcal{F}(L^2\Box) \right) \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) + \mathcal{O}[R_{\mu\nu}^2] \right] = 0, \quad (3.4)$$

and thus does not suffer from the concerns of [1]. The commutator of the covariant derivative with the form factor $(1 + \mathcal{F}(L^2\Box))$ gives rise to curvature squared terms and cancels against $\mathcal{O}[R_{\mu\nu}^2]$.

4. Discussion: running coupling constants and non-locality vs acausality

One of the main mechanisms for nonlocalities of the above type is the contribution of graviton and matter loops to the quantum effective action. In quantum theory the concept of a nonlocal form factor replacing a coupling constant is not new. In fact this concept underlies the notion of the running coupling constants and sheds new light on the cosmological constant problem also from the viewpoint of the renormalization theory.

For simplicity, consider QED or Yang-Mills theory in the quadratic order in gauge field strength $F_{\mu\nu}^2$. The transition from classical to quantum effective action, $S \rightarrow S_{\text{eff}}$, boils down to the replacement of the local invariant by

$$\frac{1}{g^2} \int dx F_{\mu\nu}^2 \rightarrow \int dx F_{\mu\nu} g_{\text{eff}}^{-2}(-\Box) F^{\mu\nu}. \quad (4.1)$$

Here the effective coupling constant $g_{\text{eff}}^{-2}(-\square)$ is actually a nonlocal form factor playing the role of $\mathcal{F}(L^2\square)$ above. It is given in terms of the renormalized running coupling $g_R^2(\mu^2)$ and the beta-function β and reads, say in the one-loop approximation, as

$$\frac{1}{g_{\text{eff}}^2(-\square)} = \frac{1}{g_R^2(\mu^2)} + \beta \ln \left(\frac{-\square}{\mu^2} \right), \quad (4.2)$$

where μ^2 is an auxiliary dimensional parameter. The form factor $1/g_{\text{eff}}^2(-\square)$ is independent of this parameter in virtue of the renormalization group equation for $g_R^2(\mu^2)$. Actually, this serves as a basis for the folklore statement that μ^2 determines the energy scale of the problem – quantum effects reduce to the classical effects with the bare coupling constant g replaced by the running one $g_R(\mu^2)$ at $\mu^2 = -\square$ (this formal substitution in the Euclidean domain of the form factor (4.2) annihilates its nonlocal logarithmic part and, thus, serves as a qualitative justification for such an interpretation). Vicy versa, the knowledge of $g_R(\mu^2)$ as a solution of the RG equation allows one to recover the corresponding nonlocal part of the effective action.

This concept, though being extremely fruitful in context of gauge field theory, fails when applied to the gravitational theory in the sector of the cosmological and Einstein-Hilbert terms⁴. Indeed, naive replacement of ultralocal cosmological and gravitational coupling constants by nonlocal form factors

$$\int dx g^{1/2} \left(\Lambda - M_P^2 R \right) \rightarrow \int dx g^{1/2} \left(\Lambda(\square) - M_P^2(\square) R \right) \quad (4.3)$$

is meaningless because the action of the covariant d'Alembertian on the right hand side always picks up its zero mode, and both form factors reduce to its numerical values in far infrared, $\Lambda(0)$, $M_P^2(0)$. This happens because the argument of $\Lambda(\square)$ has nothing to act upon but 1 (or $g^{1/2}$), and with $M_P^2(\square) R$ the same happens after integration by parts [15]. Therefore, even if one has solutions of renormalization group equations $\Lambda_R(\mu^2)$, $(M_P^2)_R(\mu^2)$, like those obtained in [13, 14], one cannot automatically recover the corresponding pieces of effective action or the corresponding nonlocal correlation functions.

The construction of Sects. 2 and 3 above suggests that the running coupling constant “delocalization” of M_P^2 should be done in the (already nonlocal) representation of the Einstein action (2.7). It is important that its curvature expansion begins with the quadratic order. Therefore, it explicitly allows one to insert the nonlocal form factor $M_P^2(\square)$ *between two curvatures* so that no integration by parts would result in its degeneration to a trivial constant. It would be interesting to see how a similar mechanism

⁴When applied to formally renormalizable (albeit non-unitary) curvature-squared gravitational models [13] or models with generalized renormalization group for infinite number of charges [14].

works for the nonlocal cosmological “constant” $\Lambda(\square)$.⁵ Nontrivial mechanisms of its generation due to infrared asymptotics of the effective action, or late-time asymptotics of the corresponding heat kernel, are discussed in [18].

Nonlocalities of the type (3.3) also arise in a certain class of braneworld models [8, 9, 3]. They cannot appear in models of the Randall-Sundrum type with strictly localized zero modes, because in these models nontrivial form factors basically arise in the transverse-traceless sector of the action (as kernels of nonlocal quadratic forms in *Weyl* tensor [12]). In contrast to these models, the nonlocal part of (3.3) is not quadratic in the Weyl tensor, $\int dx g^{1/2} W_{\mu\nu\alpha\beta}^2 \sim \int dx g^{1/2} (R_{\mu\nu}^2 - \frac{1}{3}R^2)$ (with the insertion of a nonlocal form factor between the curvatures). Rather, (3.3) includes the structure $\int dx g^{1/2} (R_{\mu\nu}^2 - \frac{1}{2}R^2)$ which contains the *conformal* sector. It is this sector which is responsible for the potential resolution of the cosmological constant problem. It becomes dynamical in models with metastable graviton like the Gregory-Rubakov-Sibiryakov model [8] or Dvali-Gabadaze-Porrati model (DGP) [9]. In particular, for the (4+1)-dimensional DGP model the (Euclidean) form factor $\mathcal{F}(L^2\square)$ is singular at $\square \rightarrow 0$ and has the form [9, 21, 22]

$$M_P^2 \mathcal{F}(L^2\square) = \frac{M^3}{\sqrt{-\square}}, \quad M^3 = \frac{M_P^2}{L}, \quad (4.4)$$

where $M \sim 10^{-21} M_P \sim 100$ MeV is a mass scale of the bulk gravity as opposed to the Planckian scale of the Einstein term on the brane $M_P \sim 10^{19}$ GeV.

Both form factors (4.2) and (4.4) are unambiguously defined only in the Euclidean space where the d'Alembertian \square is negative semi-definite. This raises the problem of their continuation to the physical spacetime where the issues of causality and unitarity become important. The principles of this continuation depend on the physical origin of nonlocality in $\mathcal{F}(L^2\square)$. Depending on whether it has a quantum nature like in (4.2) or brane induced nature like in (4.4) these principles can range from the usual Wick rotation to such currently developing paradigms as holographic dS/CFT-conjecture [19], the concept of time as a holographically generated dimension, etc. [1]. Let us first discuss nonlocal form factors of quantum nature.

Scattering problems for in-out matrix elements of quantum field $\hat{\varphi}$, $\langle out | \hat{\varphi} | in \rangle$, in spacetime with asymptotically-flat past and future imply a usual Wick rotation. The expectation-value problem or the problem for in-in mean value of the quantum

⁵On dimensional grounds one should expect that a quadratic action modelling the cosmological term would read as $\sim \Lambda \int dx g^{1/2} R^{\mu\nu} (1/\square^2) R_{\mu\nu}$ – the structure modifying (2.7) by one extra power of \square in the denominator. This structure (also suggested in [15] and discussed within the renormalization group theory) appears in two-brane models [12] and as a covariant completion of the mass term in models of massive gravitons [16] and numerous discussions of the van Dam-Veltman-Zakharov discontinuity [17].

field, $\phi = \langle in | \hat{\phi} | in \rangle$, is more complicated and incorporates the Schwinger-Keldysh diagrammatic technique [20]. In this technique the effective equations for ϕ cannot be obtained as variational derivatives of some action functional⁶. So for this problem the action as a source of effective equations does not exist at all. However, there exists a special case of the quantum initial data in the form of the *Poincare-invariant in-vacuum* in asymptotic past, $|in\rangle = |in, vac\rangle$. Effective equations for ϕ in this vacuum can be obtained by the following procedure [4]. Calculate the *Euclidean* effective action in asymptotically-flat spacetime, take its variational derivative containing the nonlocal form factors which are uniquely specified by zero boundary conditions at Euclidean infinity. Then formally go over to the Lorentzian spacetime signature with the *retardation* prescription for all nonlocal form factors. These retarded boundary conditions uniquely specify the nonlocal effective equations and guarantee their causality. This procedure was proven in [4] and also put forward in a recent paper [23] as the basis of the covariant nonlocal model of MOND theory [24].

As we see, this procedure justifies the Euclidean setup used above and suggests straightforward applications in the expectation-value problem of the above type. Interestingly, in this setting no contradiction arises between the nonlocal nature of the *Euclidean* action and *causal* nature of nonlocal equations of motion in Lorentzian spacetime. In this respect the situation is essentially different from the assumptions of [1] where acausality of equations of motion is necessarily attributed to the nonlocal action. In fact, this property requires a detailed analysis of why only the phenomena slow at the cosmological scale L turn out to be acausal, while the phenomena generated by “small” sources ($\ll L$) are essentially causal. No such assumptions are needed in effective equations for expectation values which are fundamentally causal despite their nonlocality. These equations have interesting applications in quantum gravitational context and, in particular, show the phenomenon of the cosmological acceleration due to infrared back-reaction mechanisms [25].

The situation with brane induced nonlocalities and their causality status is more questionable and conceptually open. For example, the branch point of the square root in the nonlocal form factor (4.4) is apparently related to different branches of cosmological solutions including the scenario of cosmological acceleration [21]. Therefore, in contrast to tentative models of [1] with finite $\mathcal{F}(0) \gg 1$, which only interpolate between two Einstein theories with different gravitational constants $G_{LD} \sim G_P/\mathcal{F}(0) \ll G_P$, the DGP model is anticipated to suggest the mechanism of the cosmological acceleration. This implies the replacement of the asymptotically-flat spacetime by the

⁶In the general case they can be obtained by varying a special two-field functional with respect to one field, the both fields subsequently being set coincident and equal to the mean field in question [20].

asymptotically-deSitter one. For small values of asymptotic curvature (as is the case of the observable horizon scale $H_0^2/M_P^2 \sim 10^{-120}$) the curvature expansion used above seems plausible, although the effect of the asymptotic curvature might be in essence nonperturbative. Therefore, the above construction might have to be modified. In particular, the Gibbons-Hawking term should be replaced by its asymptotically-deSitter analogue and the expansion in powers of the curvature should be replaced by the expansion in powers of its deviation from the asymptotic value $R_{\mu\nu} - \frac{1}{d}g_{\mu\nu}R_\infty$. This would introduce in the formalism as a free parameter the value of the curvature in far future, $R_\infty \sim H_0^2$, reflecting the measure of acausality in the model. It might be related to the CFT central charge, $c_{UV} = M_P^2/R_\infty \sim 10^{120}$ [1] – the number of holographic degrees of freedom in dS/CFT conjecture. The resulting modifications in the above construction are currently under study and will be presented elsewhere.

Acknowledgements

The author would like to thank V.Mukhanov, S.Solodukhin and R.Woodard for helpful stimulating discussions and is grateful to G.Gabadadze for useful comments on the first draft of this paper. He is also grateful for hospitality of the Physics Department of LMU, Munich, where a part of this work has been done under the grant SFB375. This work was also supported by the Russian Foundation for Basic Research under the grant No 02-02-17054 and the LSS grant No 1578.2003.2.

References

- [1] N.Arkani-Hamed, S.Dimopoulos, G.Dvali and G.Gabadadze, Nonlocal modification of gravity and the cosmological constant problem, hep-th/0209227.
- [2] S.Weinberg, Rev. Mod. Phys. **61** (1989) 1.
- [3] M.K.Parikh and S.N.Solodukhin, Phys. Lett. **B503** (2001) 384, hep-th/0012231.
- [4] A.O.Barvinsky and G.A.Vilkovisky, Nucl. Phys. **B 282** (1987) 163.
- [5] A.O.Barvinsky and G.A.Vilkovisky, Nucl. Phys. **B333** (1990) 471.
- [6] A.O.Barvinsky, Yu.V.Gusev, G.A.Vilkovisky and V.V.Zhytnikov, J. Math. Phys. **35** (1994) 3525-3542; J. Math. Phys. **35** (1994) 3543-3559.
- [7] A.Adams, J.McGreevy and E.Silverstein, hep-th/0209226.

- [8] R.Gregory, V.A.Rubakov and S.M.Sibiryakov, Phys. Rev. Lett. **84** (2000) 5928, hep-th/0002072.
- [9] G.Dvali, G.Gabadadze and M.Porrati, Phys. Rev. Lett. **B485** (2000) 208, hep-th/0005016.
- [10] S.L.Dubovsky and V.A.Rubakov, Phys. Rev. **D67** (2003) 104014, hep-th/0212222.
- [11] A.O.Barvinsky, Phys. Rev. **D 65** (2002) 062003, hep-th/0107244.
- [12] A.O.Barvinsky, A.Yu.Kamenshchik, A.Rathke and C.Kiefer, Phys. Rev. **D67** (2003) 023513, hep-th/0206188.
- [13] E.S. Fradkin and A.A. Tseytlin, Phys.Lett. **B104** (1981) 377; A.O.Barvinsky and I.G.Avramidi, Phys.Lett. **B159** (1985) 269.
- [14] A.O.Barvinsky, A.Yu.Kamenshchik and I.P.Karmazin, Phys.Rev. **D48** (1993) 3677, gr-qc/9302007.
- [15] E.V.Gorbar and I.L.Shapiro, JHEP 0302 (2003) 021.
- [16] M.Porrati, Phys. Lett. **B534** (2002) 209, hep-th/0203014.
- [17] H. van Dam and M.J.Veltman, Nucl. Phys. **D22** (1970) 397; V.I.Zakharov, JETP Lett. **12** (1970) 312; M.Porrati, Phys.Lett. **B498** (2001) 92, hep-th/0011152.
- [18] A.O.Barvinsky, Yu.V.Gusev, V.F.Mukhanov and D.V.Nesterov, Nonperturbative late time asymptotics for heat kernel in gravity theory, hep-th/0306052.
- [19] A.Strominger, JHEP **0110** (2001) 034, hep-th/0106113; JHEP **0111** (2001) 049, hep-th/0110087.
- [20] J.Schwinger, J. Math. Phys. **2** (1961) 407; J.L.Buchbinder, E.S.Fradkin and D.M.Gitman, Fortschr. der Phys. **29** (1981) 187; R.D.Jordan, Phys. Rev. **D33** (1986) 444.
- [21] C.Deffayet, G.Dvali and G.Gabadadze, Phys. Rev. **D65** (2002) 044023, astro-ph/0105068.
- [22] G.Dvali, A.Gruzinov and M.Zaldarriaga, The accelerated Universe and the Moon, hep-ph/0212069.
- [23] M.E.Soussa and R.P.Woodard, A nonlocal metric formulation of MOND, astro-ph/0302030.

- [24] M.Milgrom, *Astrophys. J.* **270** (1983) 365, *Astrophys. J.* **270** (1983) 371;
J.Bekenstein and M.Milgrom, *Astrophys. J.* **286** (1984) 7.
- [25] L.R.Abramo and R.P.Woodard, *Phys. Rev.* **D65** (2002) 063516; V.K.Onemli and
R.P.Woodard, *Class.Quant.Grav.* **19** (2002) 4607, gr-qc/0204065.